Multiquark picture for Σ (1620)

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Abstract

In this work we report on a new QCD sum rule analysis to predict masses of the excited baryon states (e.g. Σ (1620) and Λ (1405)) by using multiquark interpolating fields $((q\bar{q})(qqq))$. For the Σ (1620) we consider the $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ (I=1) multiquark interpolating fields. The calculated mass from those multiquark states is about 1.592 GeV. For the Λ (1405) we first show the result using the $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ (I=0) multiquark interpolating field, and compare the calculated mass to that of our previous result using the $\pi^0\Sigma^0$ multiquark state. We then show that the mass 1.405 GeV is well reproduced when using the $\bar{K}N$ (I=0) multiquark state. The uncertainties in our sum rules are also discussed.

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1. INTRODUCTION

QCD sum rule [1–3] is a powerful tools to extract various properties of hadrons. However, most QCD sum rule approaches have been applied to the lowest lying states, and it is rather difficult to properly extract physical properties of the excited states. For example, there have been only a limited number of works on the excited baryons [4–8] in which we are interested in this work.

Recently, we have proposed a new QCD sum rule analysis [9] for calculating the mass of the Λ (1405). It was based on using the multiquark interpolating field $((q\bar{q})(qqq))$ instead of the usual nucleon three quark interpolating field (qqq).

In the case of the Λ (1405) its nature is not revealed completely yet, i.e. whether it is an ordinary three-quark state or a $\bar{K}N$ bound state or a mixed state of the previous two possibilities [10]. In Ref. [9] we have focused on the decay channel of the Λ (1405) and introduced the $\pi^0\Sigma^0$ multiquark interpolating field in order to get the Λ (1405) mass since the Λ (1405) is only observed in the mass spectrum of the $\pi\Sigma$ channel (I=0). It has been found that the multiquark picture can be used to extract physical properties of the excited baryons; e.g. the mass of the excited baryon which is not fully accessible in the conventional QCD sum rule approach.

In this work we extend our previous analysis to the isospin I=1 multiquark states, i.e. $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ multiquark states. Our interpolating fields couple to both positive- and negative-parity, spin-$\frac{1}{2}$ baryon states. Among the Σ particles (I=1), the lowest spin-$\frac{1}{2}$ state which couples to the $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ channels is the Σ (1620) although the evidence of its existence is only fair [10]. We do not know the genuine structure of the Σ (1620). However, we can construct possible three multiquark states for the Σ (1620) considering its decay channels. Then, we can obtain the Σ (1620) mass by following the same procedures in Ref. [9], i.e. by comparing the mass of the $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ multiquark states each other.

In Sec.2 we present QCD sum rules for the I=1 multiquark states and explain how to get the Σ (1620) mass. In Sec. 3 we also present a QCD sum rule for the Λ (1405) mass by taking into account the $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ multiquark interpolating field, and compare it with the previous result for the $\pi^0\Sigma^0$ multiquark interpolating field. We discuss the uncertainties in our sum rules and summarize our results in Sec.4.

2. QCD SUM RULES FOR I=1 MULTIQUARK STATES

Let's consider the following correlator:

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle T(J(x)\bar{J}(0))\rangle, \tag{1}$$

where $J = \pi^+\Sigma^- - \pi^-\Sigma^+$ or $J = \bar{K}^0 n - K^- p$, or $J = \pi\Lambda$ correspond to the multiquark interpolating fields for the isospin I=1 states. The overall factor is irrelevant in our calculation. Here, we take the interpolating fields for the nucleon, the Σ, and the Λ particle as usual ones in the QCD sum rule calculations [11,2]. For example, in the case of the $\pi^0\Lambda$ multiquark state we take $J = \epsilon_{abc}(\bar{u}_e i\gamma^5 u_e - \bar{d}_e i\gamma^5 d_e)([u_a^T C\gamma_\mu s_b]\gamma^5\gamma^\mu d_c - [d_a^T C\gamma_\mu s_b]\gamma^5\gamma^\mu u_c)$, where u, d and s are the up, down and strange quark fields, and a, b, c, e are color indices. T denotes the transpose in Dirac space and C is the charge conjugation matrix.

The conventional QCD sum rule approach shows that the continuum effect becomes larger with increasing the dimension of the interpolating field and thus the results are more sensitive to a continuum threshold. However, as shown below we suggest a new approach to get the Σ (1620) mass regardless of the large continuum effect.

In the case of the $\pi^+\Sigma^- - \pi^-\Sigma^+$ and $\bar{K}^0 n - K^- p$ multiquark interpolating fields there are no exchange diagrams such as Fig. 1(a), where the lowest two lines correspond to the quark fields of a meson and the others are those of a baryon. Then, for example, in the case of the $\bar{K}N$ multiquark states the mass of the I=1 state is the same as that of the I=0 state, i.e. the $\bar{K}^0 n + K^- p$ multiquark state. Hence, for the $\bar{K}N$ and $\pi\Sigma$ (I=1) multiquark states we use the $K^- p$ and $\pi^-\Sigma^+$ multiquark sum rules in the previous work [9].

On the other hand, in the case of the $\pi\Lambda$ multiquark states both the $\pi^0\Lambda$ and $\pi^\pm\Lambda$ multiquark interpolating fields give the same mass within SU(2) symmetry (i.e. $m_u = m_d = 0$ and $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle$). Thus, in what follows we present a QCD sum rule for the $\pi^0\Lambda$ multiquark state only. The OPE side has two structures:

$$\Pi^{OPE}(q^2) = \Pi_q^{OPE}(q^2)\not{q} + \Pi_1^{OPE}(q^2)\mathbf{1}. \tag{2}$$

In this paper, however, we only present the sum rule from the Π_1 structure (hereafter referred to as the Π_1 sum rule) because the Π_1 sum rule (the chiral-odd sum rule) is generally more reliable than the Π_q sum rule (the chiral-even sum rule) as emphasized in Ref. [12] and also in our previous work [9]. The OPE side is given as follows.

$$\begin{aligned}
\Pi_1^{OPE}(q^2) = &+ \frac{11\ m_s}{\pi^8\ 2^{18}\ 3^2\ 5^2} q^{10} ln(-q^2) + \frac{1}{\pi^6\ 2^{15}\ 3^2\ 5}(40\langle\bar{q}q\rangle - 11\langle\bar{s}s\rangle)q^8 ln(-q^2) \\
&- \frac{m_s^2}{\pi^6\ 2^{14}\ 3^2}(80\langle\bar{q}q\rangle + 11\langle\bar{s}s\rangle)q^6 ln(-q^2) \\
&- \frac{m_s}{\pi^4\ 2^9\ 3^2}(23\langle\bar{q}q\rangle^2 - 20\langle\bar{q}q\rangle\langle\bar{s}s\rangle)q^4 ln(-q^2) \\
&+ \frac{1}{\pi^2\ 2^6\ 3^2}(40\langle\bar{q}q\rangle^3 + 45\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle)q^2 ln(-q^2) \\
&- \frac{m_s^2}{\pi^2\ 2^6\ 3^2}(26\langle\bar{q}q\rangle^3 - 45\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle) ln(-q^2) \\
&- \frac{m_s}{2^4\ 3^3}(132\langle\bar{q}q\rangle^4 - 37\langle\bar{q}q\rangle^3\langle\bar{s}s\rangle)\frac{1}{q^2},
\end{aligned} \tag{3}$$

where m_s is the strange quark mass and $\langle\bar{q}q\rangle$, $\langle\bar{s}s\rangle$ are the quark condensate and the strange quark condensate, respectively. Here, we let $m_u = m_d = 0 \neq m_s$ and $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle \equiv \langle\bar{q}q\rangle \neq \langle\bar{s}s\rangle$. We neglect the contribution of gluon condensates and concentrate on tree diagrams such as Figs. 1(a) and 1(b) (hereafter referred to as "bound" diagrams and "unbound" diagrams, respectively), and assume the vacuum saturation hypothesis to calculate quark condensates of higher dimensions. Note that only some typical diagrams are shown in Fig. 1.

The contribution of the "bound" diagrams is a $1/N_c$ correction to that of the "unbound" diagrams, where N_c is the number of the colors. In Eq. (3) and in what follows we set $N_c = 3$. The "unbound" diagrams correspond to a picture that two particles are flying away without any interaction between them. In the $N_c \to \infty$ limit only the "unbound" diagrams

contribute to the $\pi\Lambda$ multiquark sum rule. Then, the $\pi\Lambda$ multiquark mass should be the sum of the pion and the Λ mass in this limit. For the sake of reference, we present the OPE side in the $N_c \to \infty$ limit in the below.

$$\begin{aligned}
\Pi_1^{OPE(N_c\to\infty)}(q^2) = & + \frac{m_s}{\pi^8\ 2^{16}\ 3\ 5^2} q^{10} ln(-q^2) + \frac{1}{\pi^6\ 2^{13}\ 3\ 5}(4\langle\bar{q}q\rangle - \langle\bar{s}s\rangle) q^8 ln(-q^2) \\
& - \frac{m_s^2}{\pi^6\ 2^{12}\ 3}(8\langle\bar{q}q\rangle + \langle\bar{s}s\rangle) q^6 ln(-q^2) \\
& - \frac{m_s}{\pi^4\ 2^8\ 3^2}(19\langle\bar{q}q\rangle^2 - 12\langle\bar{q}q\rangle\langle\bar{s}s\rangle) q^4 ln(-q^2) \\
& + \frac{1}{\pi^2\ 2^4\ 3}(4\langle\bar{q}q\rangle^3 + 5\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle) q^2 ln(-q^2) \\
& - \frac{m_s^2}{\pi^2\ 2^4\ 3}(4\langle\bar{q}q\rangle^3 - 5\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle) ln(-q^2) \\
& - \frac{m_s}{3^2}(3\langle\bar{q}q\rangle^4 - \langle\bar{q}q\rangle^3\langle\bar{s}s\rangle)\frac{1}{q^2}.
\end{aligned} \tag{4}$$

The OPE sides in Eqs. (3) and (4) have the following form:

$$\begin{aligned}
\Pi_1^{OPE}(q^2) = & \ a\ q^{10} ln(-q^2) + b\ q^8 ln(-q^2) + c\ q^6 ln(-q^2) + d\ q^4 ln(-q^2) \\
& + e\ q^2 ln(-q^2) + f\ ln(-q^2) + g\ \frac{1}{q^2},
\end{aligned} \tag{5}$$

where $a, b, c, \cdots, g$ are constants. Then we parameterize the phenomenological side as

$$\frac{1}{\pi} Im\Pi_1^{Phen}(s) = \lambda^2 m\delta(s - m^2) + [-a\ s^5 - b\ s^4 - c\ s^3 - d\ s^2 - e\ s - f]\theta(s\ -\ s_0), \tag{6}$$

where m is the $\pi\Lambda$ multiquark mass and s_0 the continuum threshold. λ is the coupling strength of the interpolating field to the physical Σ (1620) state. After Borel transformation the mass m is given by

$$\begin{aligned}
m^2 = & \ M^2 \times \\
& \{ -720a(1-\Sigma_6) - \frac{120b}{M^2}(1-\Sigma_5) - \frac{24c}{M^4}(1-\Sigma_4) \\
& - \frac{6d}{M^6}(1-\Sigma_3) - \frac{2e}{M^8}(1-\Sigma_2) - \frac{f}{M^{10}}(1-\Sigma_1)\} \ / \\
& \{ -120a(1-\Sigma_5) - \frac{24b}{M^2}(1-\Sigma_4) - \frac{6c}{M^4}(1-\Sigma_3) \\
& - \frac{2d}{M^6}(1-\Sigma_2) - \frac{e}{M^8}(1-\Sigma_1) - \frac{f}{M^{10}}(1-\Sigma_0) - \frac{g}{M^{12}}\},
\end{aligned} \tag{7}$$

where

$$\Sigma_i = \sum_{k=0}^{i} \frac{s_0^k}{k\ !\ (M^2)^k}\ e^{-\frac{s_0}{M^2}}. \tag{8}$$

Fig. 2 shows the Borel-mass dependence of the $\pi\Lambda$ multiquark mass at $s_0 = 2.756$ GeV^2 taken by considering the next Σ (1660) [10]. There is a plateau for the large Borel mass,

but this is a trivial result from our crude model on the phenomenological side. Hence we do not take this as the $\pi\Lambda$ multiquark mass and neither as the Σ (1620) mass.

Instead, we draw the Borel-mass dependence of the coupling strength λ^2 at $s_0 = 2.756$ GeV^2 in Fig. 3. There is the maximum point in the figure. It means that the $\pi\Lambda$ multiquark interpolating fields couples strongly to the physical Σ (1620) state at this point. Then we take the Σ (1620) mass as that of the $\pi\Lambda$ multiquark state at the point. However, s_0 is taken by hand considering the experimental Σ (1660) mass. It would be better to determine an effective threshold s_0 from the present sum rule itself. In what follows we explain how to determine the effective threshold and thus the Σ (1620) mass.

The Σ (1620) is the lowest spin-$\frac{1}{2}$ state which couples to the three I=1 multiquark states, i.e. the $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ states. The aim of the present work is to get the masses of the $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ multiquark states, where the coupling strength of each multiquark state has its maximum value. We take the same threshold for each multiquark sum rule. Why we can compare the multiquark masses at the same threshold although they are obtained with different interpolating fields will be clear later. We choose the threshold in order that the $\bar{K}N$ multiquark mass becomes the sum of the kaon and the nucleon mass at least. Then, above the threshold the Σ (I=1) particle can couple to the $\pi\Sigma$, $\pi\Lambda$ and $\bar{K}N$ multiquark states, while below the threshold to the $\pi\Sigma$ and/or $\pi\Lambda$ multiquark state(s) only.

Let us consider the $\bar{K}N$ multiquark sum rule to get the effective threshold. The OPE side is the same as that of the K^+p multiquark sum rule without the contribution of "bound" diagrams [9], and thus given by

$$\begin{aligned}\Pi_1^{OPE(N_c\to\infty)}(q^2) = & + \frac{1}{\pi^6\ 2^{13}\ 3\ 5}\langle\bar{q}q\rangle q^8 ln(-q^2) - \frac{m_s^2}{\pi^6\ 2^{11}\ 3}\langle\bar{q}q\rangle q^6 ln(-q^2) \\ & - \frac{m_s}{\pi^4\ 2^8\ 3}(2\langle\bar{q}q\rangle^2 - \langle\bar{q}q\rangle\langle\bar{s}s\rangle)q^4 ln(-q^2) \\ & + \frac{1}{\pi^2\ 2^4\ 3}(2\langle\bar{q}q\rangle^3 + \langle\bar{q}q\rangle^2\langle\bar{s}s\rangle)q^2 ln(-q^2) \\ & - \frac{m_s^2}{\pi^2\ 2^4\ 3}(4\langle\bar{q}q\rangle^3 - \langle\bar{q}q\rangle^2\langle\bar{s}s\rangle)ln(-q^2) \\ & - \frac{m_s}{2\ 3^2}(2\langle\bar{q}q\rangle^4 - \langle\bar{q}q\rangle^3\langle\bar{s}s\rangle)\frac{1}{q^2}. \end{aligned} \tag{9}$$

Fig. 4 presents the Borel-mass dependence of the $\bar{K}N$ multiquark mass in the fiducial Borel interval which lies in the 30 % – 50 % criteria, i.e. the contribution of the power correction is less than 30 % and that of the continuum is less than 50 %. We define $A \equiv M^2 \times \frac{-120b}{-24b}$ and $B \equiv M^2 \times \frac{-120b - \frac{24c}{M^2} - \frac{6d}{M^4} - \frac{2e}{M^6} - \frac{f}{M^8}}{-24b - \frac{6c}{M^2} - \frac{2d}{M^4} - \frac{e}{M^6} - \frac{f}{M^8} - \frac{g}{M^{10}}}$. Then, we calculate the contribution of the power correction as $C \equiv 1 - \frac{\sqrt{A}}{\sqrt{B}}$. Using this factor we determine the lower bound of the Borel interval where C is 0.3 at most. On the other hand, we choose the upper bound of the Borel interval in order that the factor $D \equiv 1 - \frac{\sqrt{Eq.(7)}}{\sqrt{B}}$ is 0.5 at most. Since there is no plateau in the fiducial Borel interval in Fig. 4, we take the $\bar{K}N$ multiquark mass as an average value in the interval. Then, we get $s_0 = 3.852$ GeV^2 where the average mass becomes $m_K + m_p$ = 1.435 GeV.

Now, let us go back to the Σ (1620) mass. We redraw the Borel-mass dependence of the coupling strength from the $\pi\Lambda$ multiquark sum rule at $s_0 = 3.852$ GeV^2 obtained in the

above, and then take the Σ (1620) mass as the value where the coupling strength has its maximum value. In Table 1 we present the mass m for each multiquark state, where we take $\langle \bar{q}q \rangle = -\ (0.230\ \mathrm{GeV})^3$, $\langle \bar{s}s \rangle = 0.8\ \langle \bar{q}q \rangle$, and $m_s = 0.150$ GeV as input parameters. In the case of the $\bar{K}N$ and $\pi\Sigma$ multiquark states we use the same masses in Ref. [9] obtained from the K^-p and the $\pi^-\Sigma^+$ multiquark state, respectively. Fig. 5 shows the coupling strength and the mass from each multiquark state at $s_0 = 3.852\ \mathrm{GeV}^2$. The average mass from three states in the table is about 1.592 GeV, and it is rather smaller than the experimental value, 1.620 GeV [10]. Fig. 6 presents the Borel-mass dependence of the $\pi\Lambda$ multiquark mass on the strange quark mass, the strange quark condensate, and the quark condensate, respectively, at $s_0 = 3.852\ \mathrm{GeV}^2$. It seems that the SU(3) symmetry breaking effects are not significant in our sum rule.

It is interesting to note that the masses in Table 1 are very similar. We have checked that at an arbitrary threshold three multiquark sum rules give similar masses. Thus, in principle we can predict another mass of the Σ excited state using these three multiquark sum rules if the threshold is taken properly.

Let us stop here to remark why the multiquark masses are similar although they are calculated from different interpolating fields, i.e. the $\bar{K}N$, $\pi\Sigma$ and $\pi\Lambda$ multiquark interpolating fields, respectively. First of all, comparing Eq. (4) and Eq. (9) one can easily find that they have the same structure within SU(3) symmetry ($m_u = m_d = m_s = 0$, $\langle \bar{q}q \rangle = \langle \bar{s}s \rangle$). For completeness, we write down the OPE side for the $\pi\Sigma$ multiquark interpolating field in the $N_c \to \infty$ limit.

$$\begin{aligned}
\Pi_1^{OPE(N_c\to\infty)}(q^2) = & -\frac{m_s}{\pi^8\ 2^{16}\ 3\ 5^2} q^{10} ln(-q^2) + \frac{1}{\pi^6\ 2^{13}\ 3\ 5} \langle \bar{s}s \rangle q^8 ln(-q^2) \\
& + \frac{m_s^2}{\pi^6\ 2^{12}\ 3} \langle \bar{s}s \rangle q^6 ln(-q^2) - \frac{m_s}{\pi^4\ 2^7} \langle \bar{q}q \rangle^2 q^4 ln(-q^2) \\
& + \frac{1}{\pi^2\ 2^4} \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle q^2 ln(-q^2) + \frac{m_s^2}{\pi^2\ 2^4} \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle ln(-q^2) \\
& - \frac{m_s}{3^2} \langle \bar{q}q \rangle^4 \frac{1}{q^2}. \qquad (10)
\end{aligned}$$

Of course, as we said before, there is no contribution of the "bound" diagrams for the $\bar{K}N$ and $\pi\Sigma$ multiquark sum rules. Second, as shown in Fig. 6 the SU(3) symmetry breaking effects are small in our sum rules. In Fig. 7 we plot the $\pi\Lambda$ multiquark mass with(the solid line) and without(the dotted line) the SU(3) symmetry breaking effects. We draw the solid line using $\langle \bar{q}q \rangle = -\ (0.230\ \mathrm{GeV})^3$, $\langle \bar{s}s \rangle = 0.8\ \langle \bar{q}q \rangle$, and $m_s = 0.150$ GeV, while draw the dotted line taking $\langle \bar{q}q \rangle = -\ (0.230\ \mathrm{GeV})^3$, $\langle \bar{s}s \rangle = 1.0\ \langle \bar{q}q \rangle$, and $m_s = 0$. Last, the contribution of the "bound" diagrams in the $\pi\Lambda$ multiquark sum rule is very small as shown in Fig. 8. The solid line is the $\pi\Lambda$ multiquark mass from all diagrams; i.e. the "unbound" + "bound" diagrams (from Eq. (3)), while the dotted line is the mass from the "unbound" diagrams only (from Eq. (4)). Because of the above reasons the masses from three multiquark sum rules are similar. Hence, we can apply the same threshold for each multiquark sum rule and compare the multiquark mass each other.

3. QCD SUM RULES FOR I=0 MULTIQUARK STATES

In the following we get the Λ (1405) mass by taking into account the $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ (I=0) multiquark interpolating field and compare the mass with the previous result for the $\pi^0\Sigma^0$ multiquark state [9]. The $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ multiquark state is the complete basis for the I=0 state in contrast to the $\pi^0\Sigma^0$ multiquark state.

The Π_1^{OPE} for the $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ multiquark state is given by

$$\begin{aligned}
\Pi_1^{OPE}(q^2) = & -\frac{7\ m_s}{\pi^8\ 2^{18}\ 3^2\ 5} q^{10} ln(-q^2) + \frac{7}{\pi^6\ 2^{15}\ 3^2}\langle\bar{s}s\rangle q^8 ln(-q^2) \\
& + \frac{35\ m_s^2}{\pi^6\ 2^{14}\ 3^2}\langle\bar{s}s\rangle q^6 ln(-q^2) - \frac{121\ m_s}{\pi^4\ 2^9\ 3^2}\langle\bar{q}q\rangle^2 q^4 ln(-q^2) \\
& + \frac{11}{\pi^2\ 2^6}\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle q^2 ln(-q^2) - \frac{m_s^2}{\pi^2\ 2^6\ 3}(14\langle\bar{q}q\rangle^3 - 33\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle) ln(-q^2) \\
& - \frac{m_s}{2^4\ 3^3}(140\langle\bar{q}q\rangle^4 + 3\langle\bar{q}q\rangle^3\langle\bar{s}s\rangle)\frac{1}{q^2}, \qquad (11)
\end{aligned}$$

and in the $N_c \to \infty$ limit

$$\begin{aligned}
\Pi_1^{OPE(N_c\to\infty)}(q^2) = & -\frac{m_s}{\pi^8\ 2^{16}\ 5^2} q^{10} ln(-q^2) + \frac{1}{\pi^6\ 2^{13}\ 5}\langle\bar{s}s\rangle q^8 ln(-q^2) \\
& + \frac{m_s^2}{\pi^6\ 2^{12}}\langle\bar{s}s\rangle q^6 ln(-q^2) - \frac{65\ m_s}{\pi^4\ 2^8\ 3^2}\langle\bar{q}q\rangle^2 q^4 ln(-q^2) \\
& + \frac{3}{\pi^2\ 2^4}\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle q^2 ln(-q^2) - \frac{m_s^2}{\pi^2\ 2^4\ 3}(4\langle\bar{q}q\rangle^3 - 9\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle) ln(-q^2) \\
& - \frac{m_s}{3}\langle\bar{q}q\rangle^4\frac{1}{q^2}. \qquad (12)
\end{aligned}$$

In the case of the Λ (1405) it couples to the $\pi\Sigma$ channel only. Thus, we take the threshold s_0 = 3.082 GeV^2 in order that the $\pi\Sigma$ (I=0) multiquark mass becomes the sum of the pion and the Σ mass in the fiducial Borel interval when only the "unbound" diagrams are considered. Here, we use the average mass of the pions and the Σ particles, i.e. 0.138 + 1.193 = 1.331 GeV. In order to get the Borel interval we define new parameters $C' \equiv 1 - \frac{\sqrt{A'}}{\sqrt{B'}}$ and $D' \equiv 1 - \frac{\sqrt{Eq.(7)}}{\sqrt{B'}}$, where $A' \equiv M^2 \times \frac{-720a}{-120a}$ and $B' \equiv M^2 \times \frac{-720a - \frac{120b}{M^2} - \frac{24c}{M^4} - \frac{6d}{M^6} - \frac{2e}{M^8} - \frac{f}{M^{10}}}{-120a - \frac{24b}{M^2} - \frac{6c}{M^4} - \frac{2d}{M^6} - \frac{e}{M^8} - \frac{f}{M^{10}} - \frac{g}{M^{12}}}$. Following the same procedures in the previous section we get the Λ (1405) mass as shown in Table 2. The mass is very similar to that of the $\pi^0\Sigma^0$ multiquark state. In Table 2 we also present the variation of the Λ (1405) mass on the quark condensate. The mass becomes smaller as the absolute value of the quark condensate increases.

On the other hand, the $\bar{K}N$ (I=0) multiquark mass at s_0 = 3.082 GeV^2 is 1.405 GeV, and it is closer to the experimental value comparing to 1.387 GeV which was obtained previously using the threshold s_0 = 3.012 GeV^2 from the $\pi^0\Sigma^0$ multiquark sum rule [9]. Fig. 9 shows the coupling strength and the mass from the $\bar{K}N$ and $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ multiquark state, respectively.

One can easily find that within SU(3) symmetry ($m_u = m_d = m_s = 0$, $\langle\bar{q}q\rangle = \langle\bar{s}s\rangle$) Eq. (12) has the same structure as in Eq. (9). Note that the $\bar{K}N$ (I=0) multiquark sum

rule is the same as the $\bar{K}N$ (I=1) multiquark sum rule since there is no contribution of the "bound" diagrams for the $\bar{K}N$ (both I=0 and I=1) multiquark interpolating fields.

Table 3 shows the masses from the I=0 multiquark states at $s_0 = 3.852$ GeV^2, and these values correspond to the Λ (1600) mass. Because the Λ (1600) couples to both the $\bar{K}N$ and $\pi\Sigma$ channels (I=0), we get the Λ (1600) mass by comparing the $\bar{K}N$ and $\pi\Sigma$ multiquark mass at the same threshold. The average mass between the $\bar{K}N$ and $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ (I=0) multiquark states in the table is 1.601 GeV.

4. DISCUSSION

Let us discuss the uncertainties in our results. Comparing Tables 1 and 3, each average mass of the I=0 and I=1 multiquark states is slightly different from the experimental values, i.e. the Λ (1600) and the Σ (1620), respectively. Note that in the previous sections we have used the same threshold for the I=1 and I=0 $\bar{K}N$ multiquark states because we can not distinguish the I=1 state from the I=0 state in our approach.

The mass difference can be calculated by including the isospin symmetry breaking effects (i.e. $m_u \neq m_d \neq 0$, $\langle\bar{u}u\rangle \neq \langle\bar{d}d\rangle$, and electromagnetic effects) in our sum rules as in Refs. [13–20]. If this correction is included, then the threshold for the $\bar{K}N$ multiquark state will be different from the previous one. Then, the masses of other multiquark states (both the I=0 and I=1 states) can also be changed according to the new threshold.

On the other hand, one can consider the contractions between the $\bar{u}$ and u (or between the $\bar{d}$ and d) quarks in the initial state which have been excluded in our previous calculation. If this correction is taken into account, then we can distinguish between the $\bar{K}^0n+K^-p$ (I=0) and $\bar{K}^0n - K^-p$ (I=1) multiquark states because it is one of $1/N_c$ corrections. Although we are only interested in the five-quark states in the initial state, one can check easily the amount of contribution to the previous calculation. Including this correction the mass for the $\bar{K}^0n + K^-p$ (I=0) multiquark state becomes 1.590 GeV while 1.588 GeV for the $\bar{K}^0n - K^-p$ (I=1) multiquark state at $s_0 = 3.852$ GeV^2. Note that the effective threshold s_0 is obtained by including the "unbound" diagrams only and thus we can use the same threshold for both multiquark states. The masses of other multiquark states are rarely changed even if this correction is considered. It is found that this correction is very small comparing to other $1/N_c$ corrections, i.e. the contribution of "bound" diagrams. Another possibility to get the mass difference will be the correction from the possible instanton effects [21–23] to the I=0 and I=1 states, respectively.

In this work we have neglected the contribution of gluon condensates. Since we have considered the Π_1 sum rule (the chiral-odd sum rule), only the odd dimensional operators can contribute to the sum rule. For example, the contribution of the gluon condensates is given by the terms like $m_s\langle\frac{\alpha_s}{\pi}G^2\rangle$ and thus can be neglected comparing to other quark condensates of the same dimension. However, as shown in the nucleon sum rule, the gluon condensate term significantly affects the Borel-stability although it is numerically small. In this respect, further analysis including the contribution of the gluon condensates in our sum rules is required before any firm conclusions may be drawn.

In summary, the Σ (1620) mass is predicted in the QCD sum rule approach using the $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ (I=1) multiquark interpolating fields. The mass from the Π_1 sum rule (the

chiral-odd sum rule) is about 1.592 GeV. The Λ (1405) mass is also obtained considering the $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ (I=0) multiquark state. The mass is 1.424 GeV, while that of the $\bar{K}N$ (I=0) multiquark state is 1.405 GeV. On the other hand, it would be interesting to calculate the Σ (1620) mass by following the methods in Refs. [7,8] which have obtained the N (1535) and the Λ (1405) mass, respectively, using the interpolating fields with a covariant derivative.

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TABLES

TABLE 1. Mass of the $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ (I=1) multiquark states at $s_0 = 3.852$ GeV^2 ($\langle\bar{q}q\rangle =$ – (0.230 GeV)^3, $\langle\bar{s}s\rangle = 0.8\ \langle\bar{q}q\rangle$, and $m_s = 0.150$ GeV).

multiquark state	m(GeV)
$\bar{K}N$	1.589
$\pi\Sigma$	1.606
$\pi\Lambda$	1.581

TABLE 2. Mass of the $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ (I=0) multiquark state ($\langle\bar{s}s\rangle = 0.8\ \langle\bar{q}q\rangle$, $m_s =$ 0.150 GeV). [$\cdots$] means the value from the $\pi^0\Sigma^0$ multiquark state.

quark condensate (GeV^3)	s_0 (GeV^2)	m(GeV)
$-(0.210)^3$	3.093 [3.015]	1.443 [1.434]
$-(0.230)^3$	3.082 [3.012]	1.424 [1.419]
$-(0.250)^3$	3.077 [3.008]	1.409 [1.404]

TABLE 3. Mass of the $\bar{K}N$ and $\pi\Sigma$ (I=0) multiquark states at $s_0 = 3.852$ GeV^2 ($\langle\bar{q}q\rangle =$ – (0.230 GeV)^3, $\langle\bar{s}s\rangle = 0.8\ \langle\bar{q}q\rangle$, and $m_s = 0.150$ GeV). [$\cdots$] means the value from the $\pi^0\Sigma^0$ multiquark state.

multiquark state	m(GeV)
$\bar{K}N$	1.589
$\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$	1.612 [1.625]

FIGURES

FIG. 1. Diagrams. Solid lines are the quark propagators. (a) "bound" diagrams (b) "unbound" diagrams.

FIG. 2. The Borel-mass dependence of the $\pi\Lambda$ multiquark mass at $s_0 = 2.756$ GeV^2.

FIG. 3. The Borel-mass dependence of the coupling strength λ^2 from the $\pi\Lambda$ multiquark sum rule at $s_0 = 2.756$ GeV^2.

FIG. 4. The Borel-mass dependence of the $\bar{K}N$ multiquark mass in the fiducial Borel interval at $s_0 = 3.852$ GeV^2.

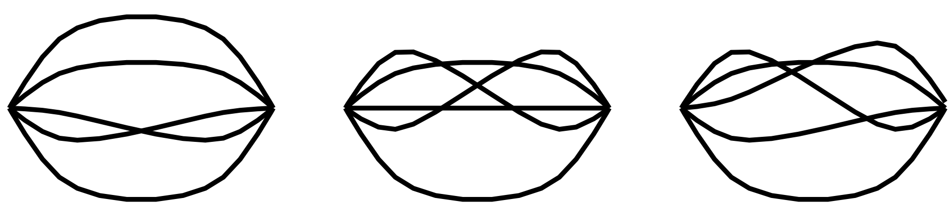
FIG. 5. The coupling strengths and masses from the $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ multiquark states at $s_0 = 3.852$ GeV^2.

FIG. 6. The Borel-mass dependence of the $\pi\Lambda$ multiquark mass at $s_0 = 3.852$ GeV^2 on (a) the strange quark mass (b) the strange quark condensate (c) the quark condensate.

FIG. 7. The SU(3) symmetry breaking effects in the $\pi\Lambda$ multiquark sum rule at $s_0 = 3.852$ GeV^2.

FIG. 8. The contribution of "bound" diagrams in the $\pi\Lambda$ multiquark sum rule at $s_0 = 3.852$ GeV^2.

FIG. 9. The coupling strengths and masses from the $\bar{K}N$ and $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$ multiquark states at $s_0 = 3.082$ GeV^2.



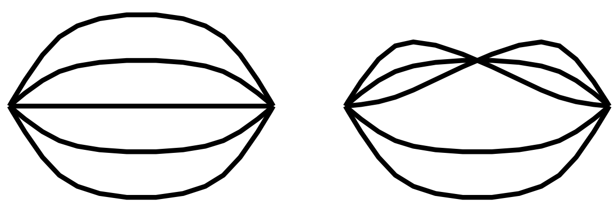
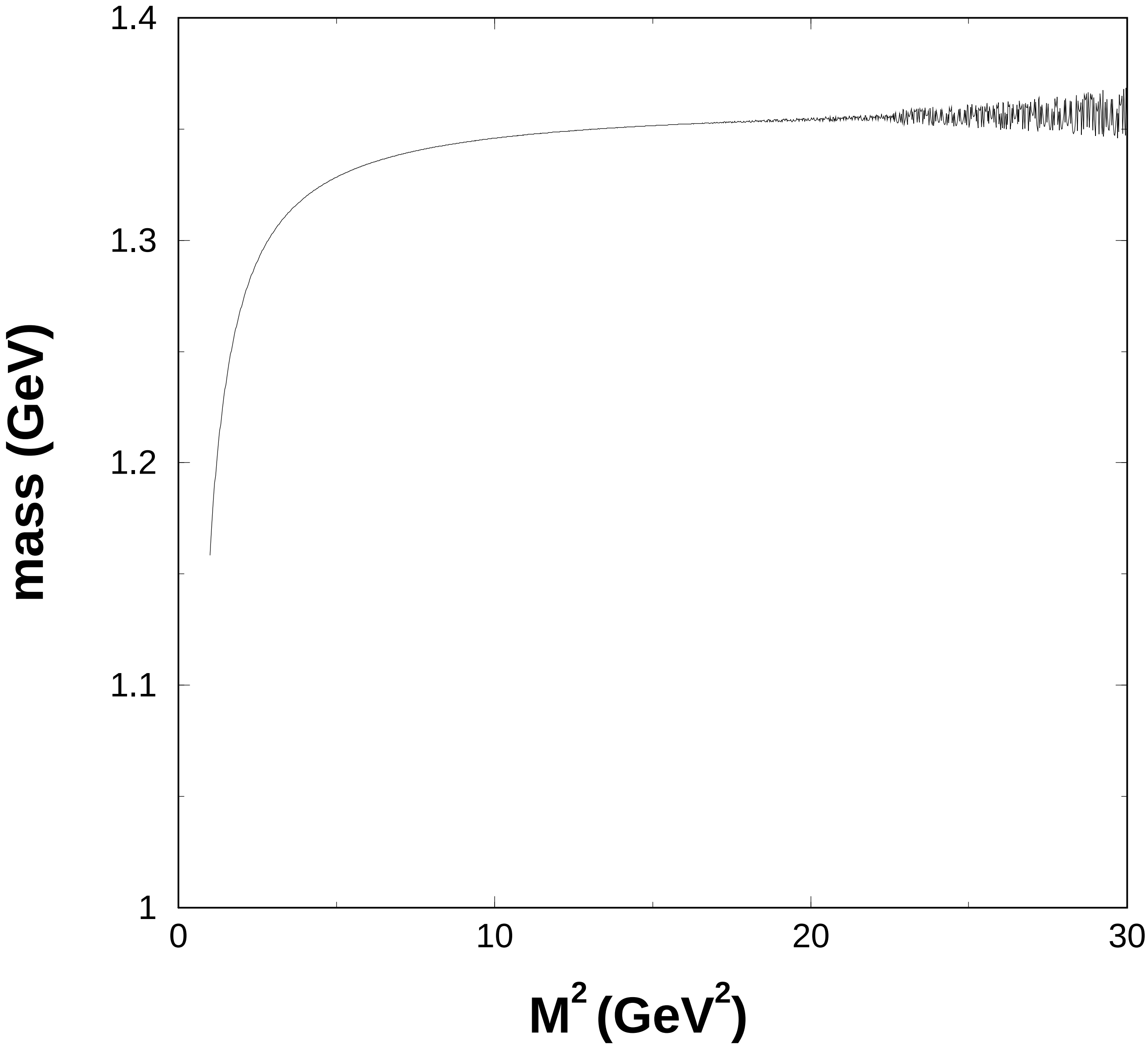


Fig. 2



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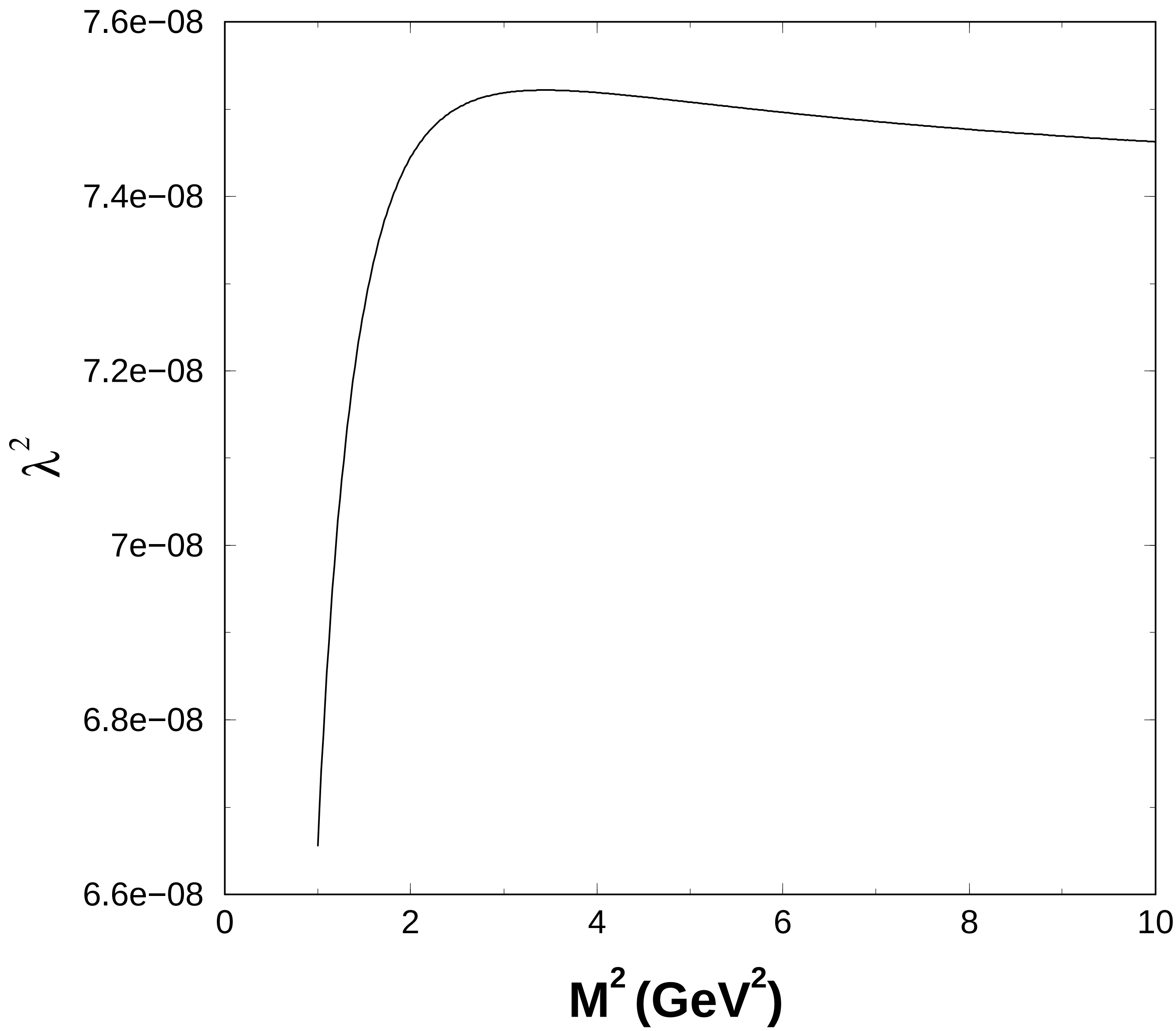

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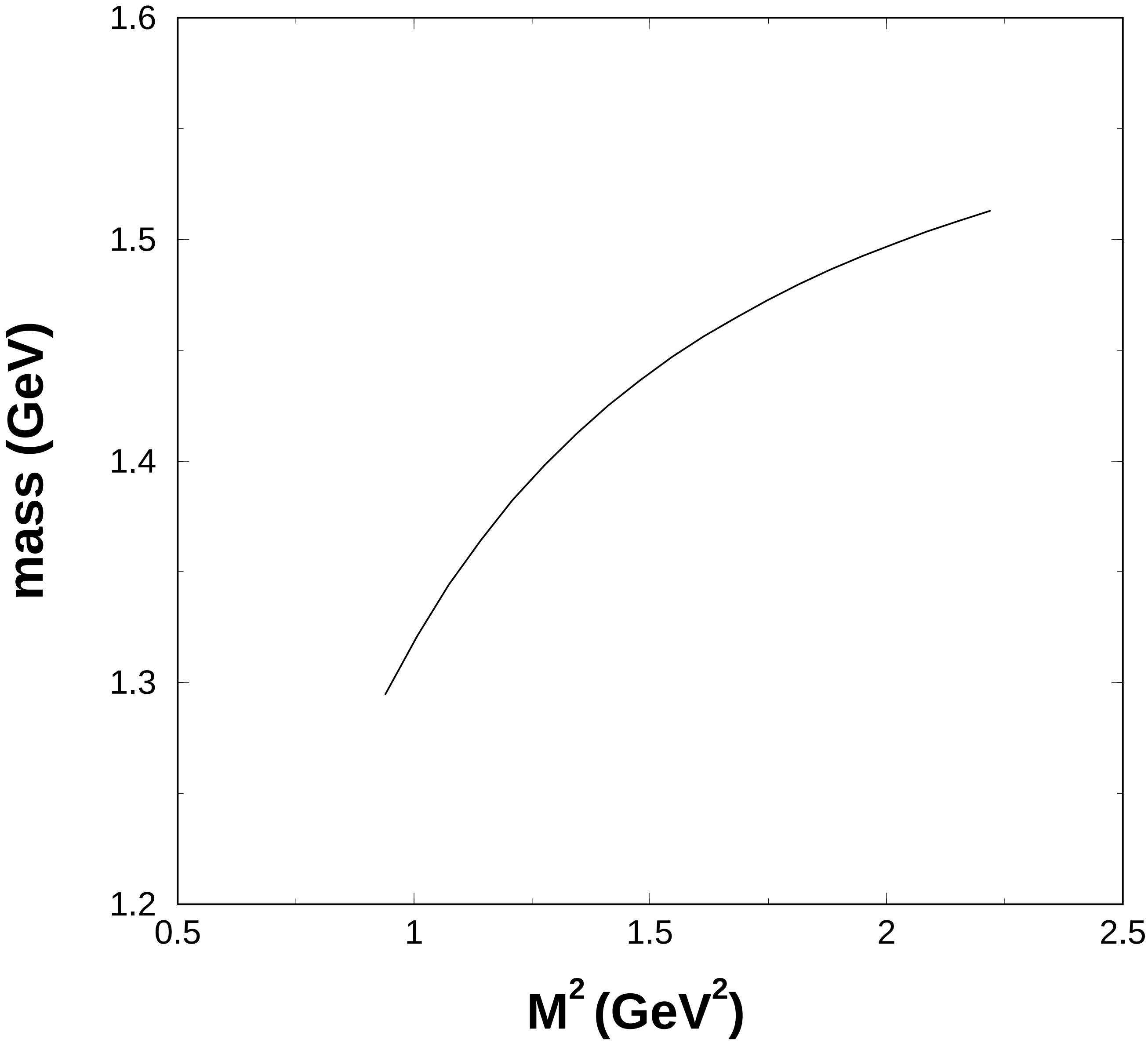

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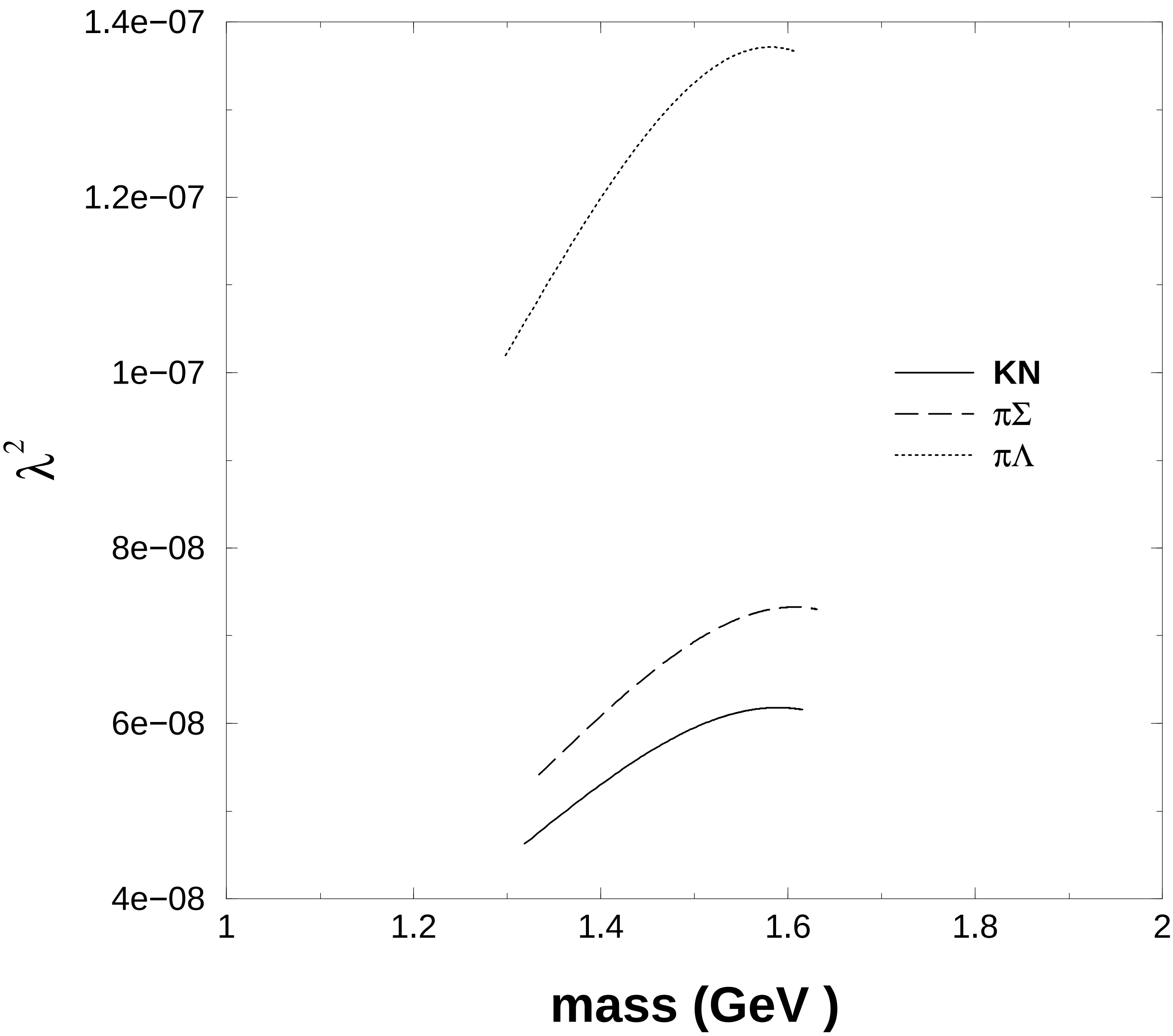

1.4e−07
1.2e−07
1e−07
8e−08
6e−08
4e−08
1
1.2
1.4
1.6
1.8
2
λ^2
KN
$\pi\Sigma$
$\pi\Lambda$
mass (GeV)

Fig. 6(a)

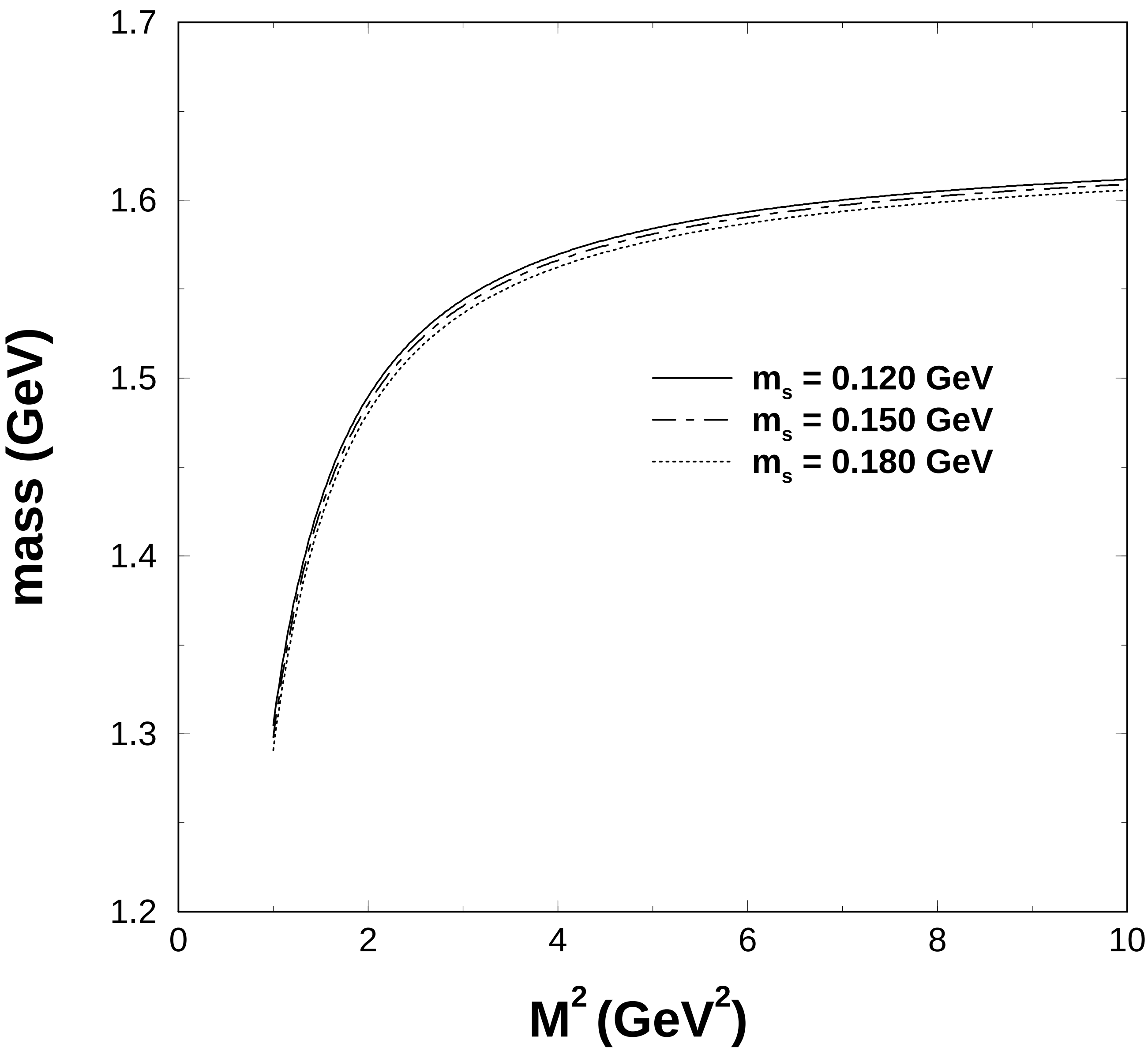
1.7
1.6
1.5
1.4
1.3
1.2
0
2
4
6
8
10
mass (GeV)
M^2 (GeV^2)
m_s = 0.120 GeV
m_s = 0.150 GeV
m_s = 0.180 GeV

Fig. 6(b)

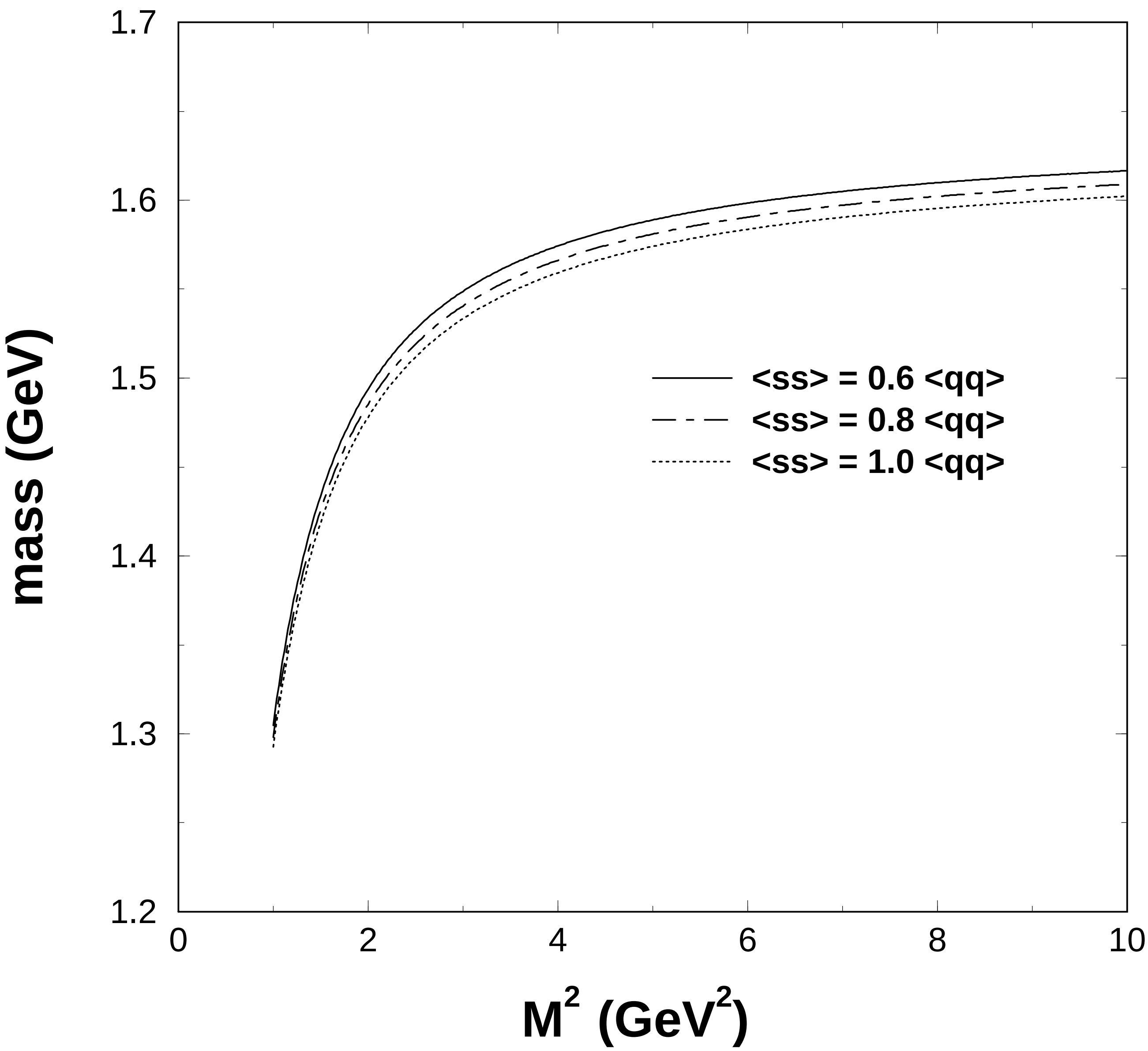
1.7
1.6
1.5
1.4
1.3
1.2
0
2
4
6
8
10
mass (GeV)
M2 (GeV2)
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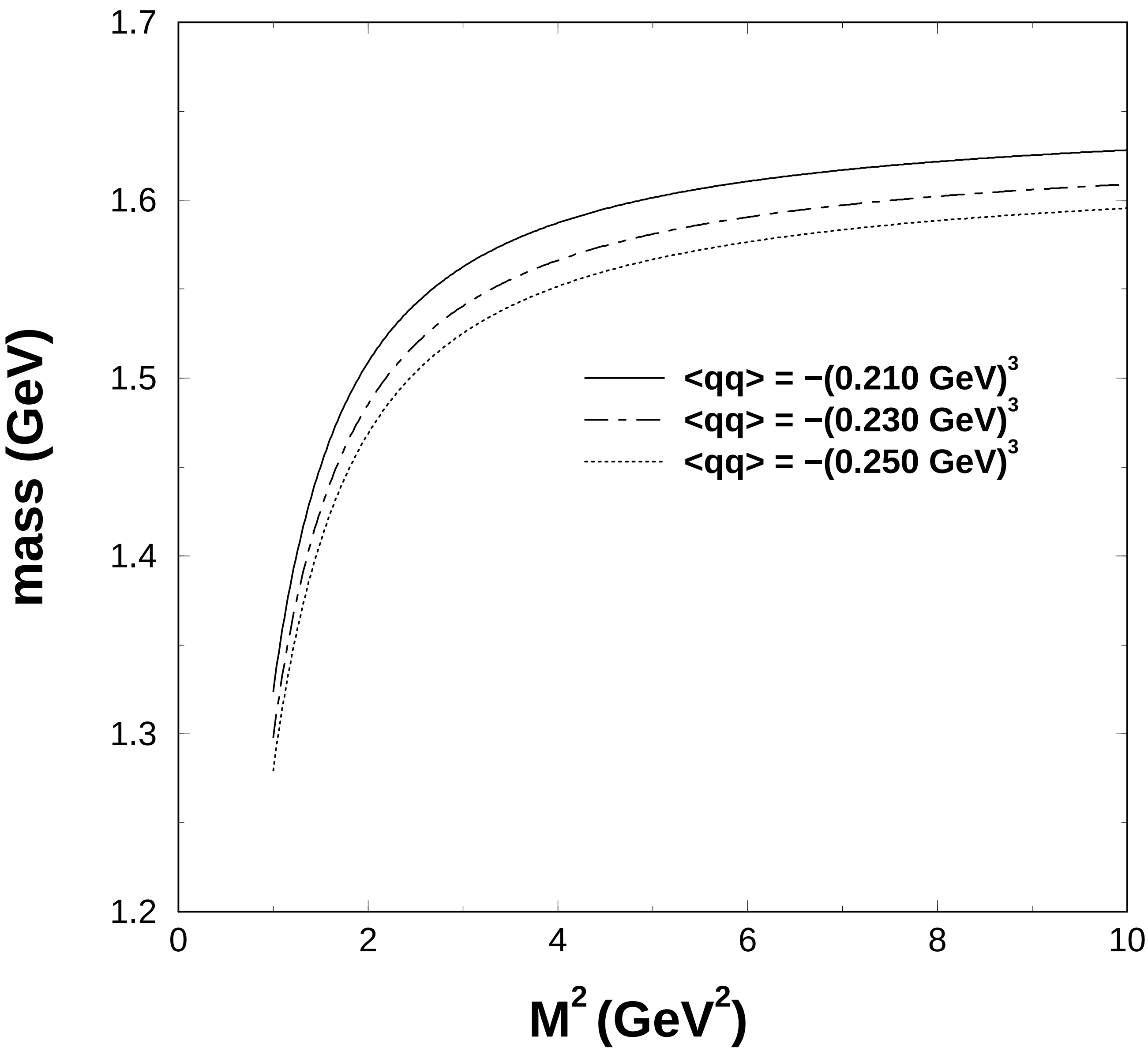
Fig. 6(c)



1.7
1.6
1.5
1.4
1.3
1.2
0
2
4
6
8
10
mass (GeV)
M^2 (GeV^2)
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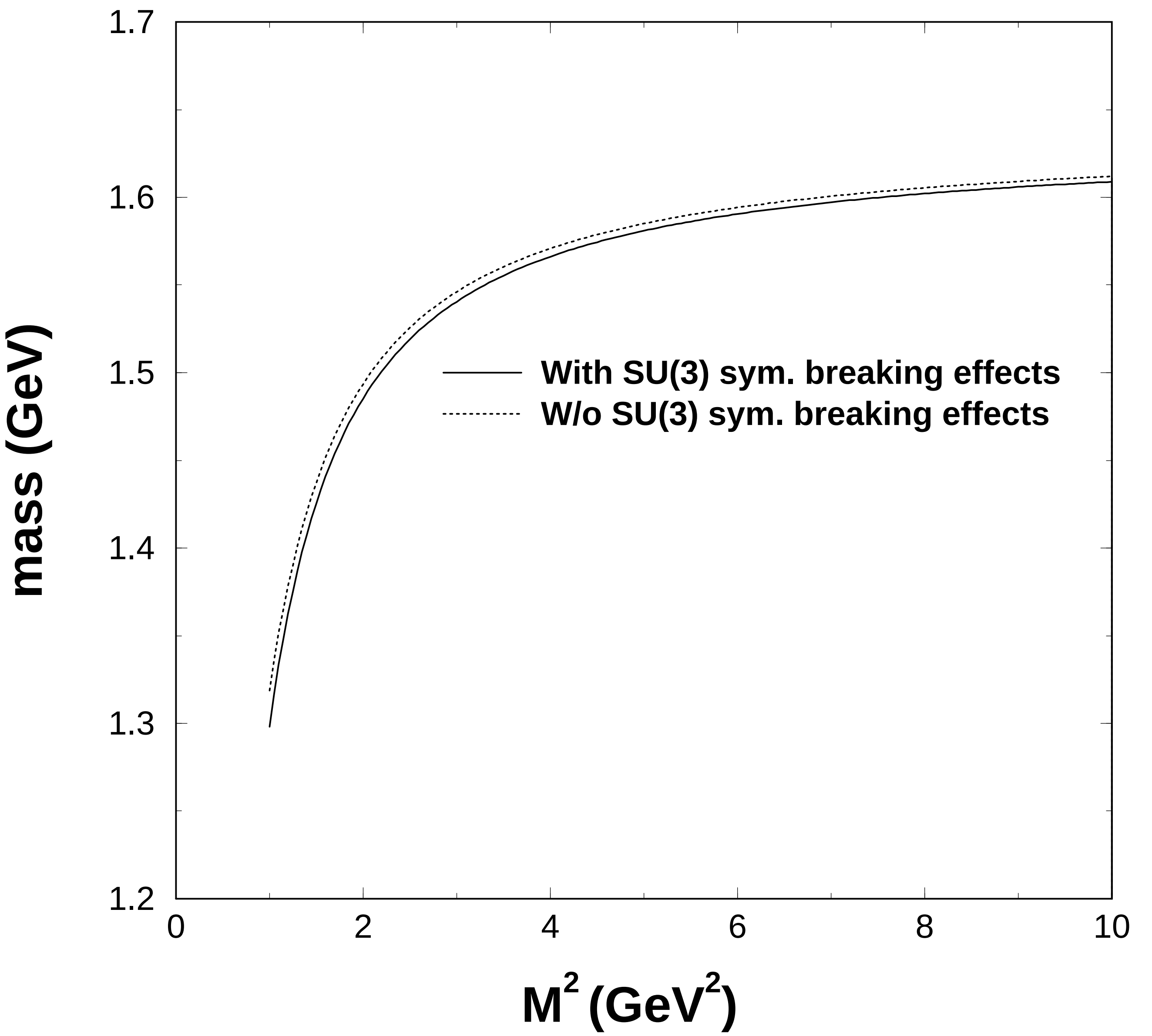
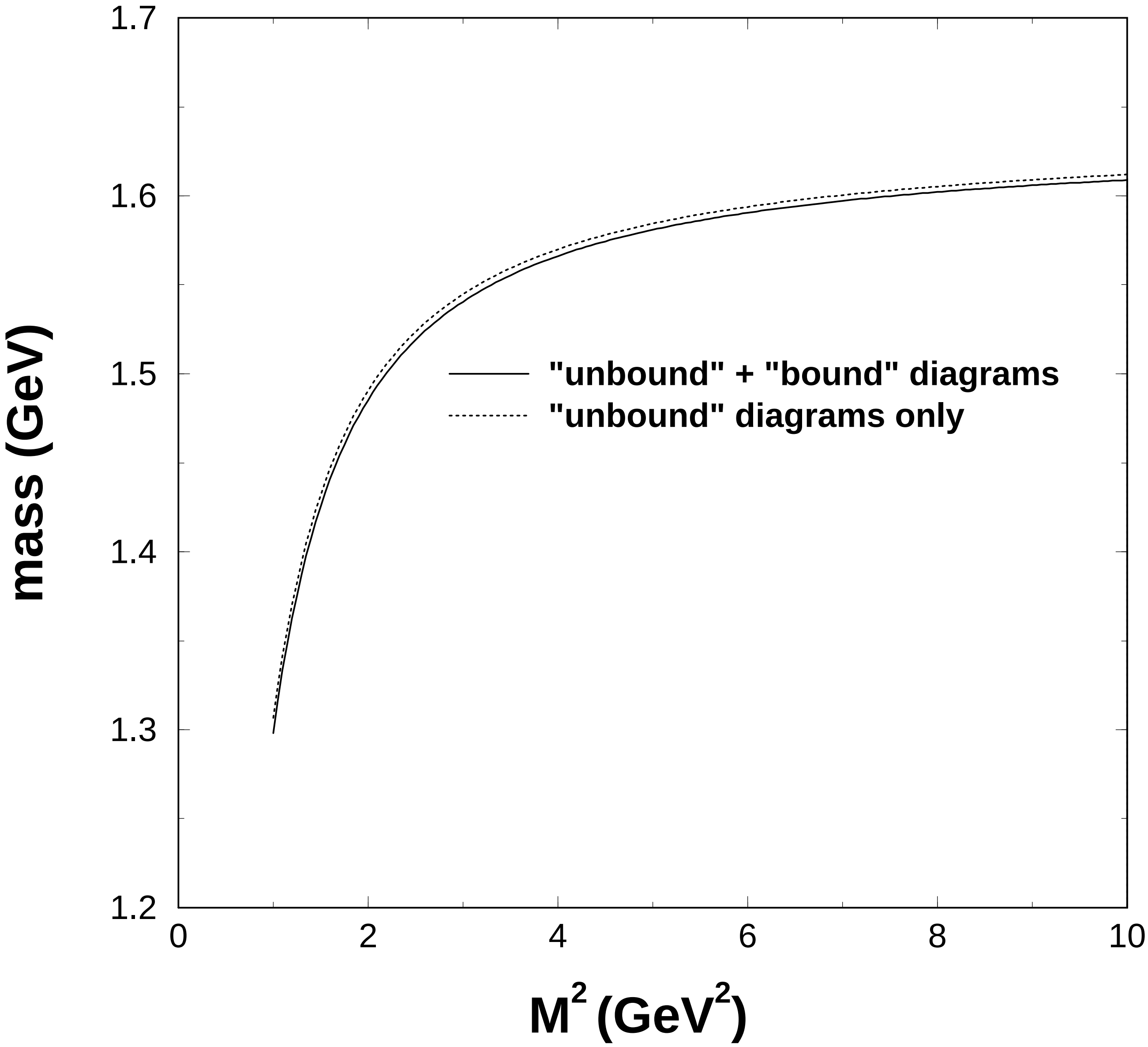
<qq> = −(0.230 GeV)^3
<qq> = −(0.250 GeV)^3

Fig. 7



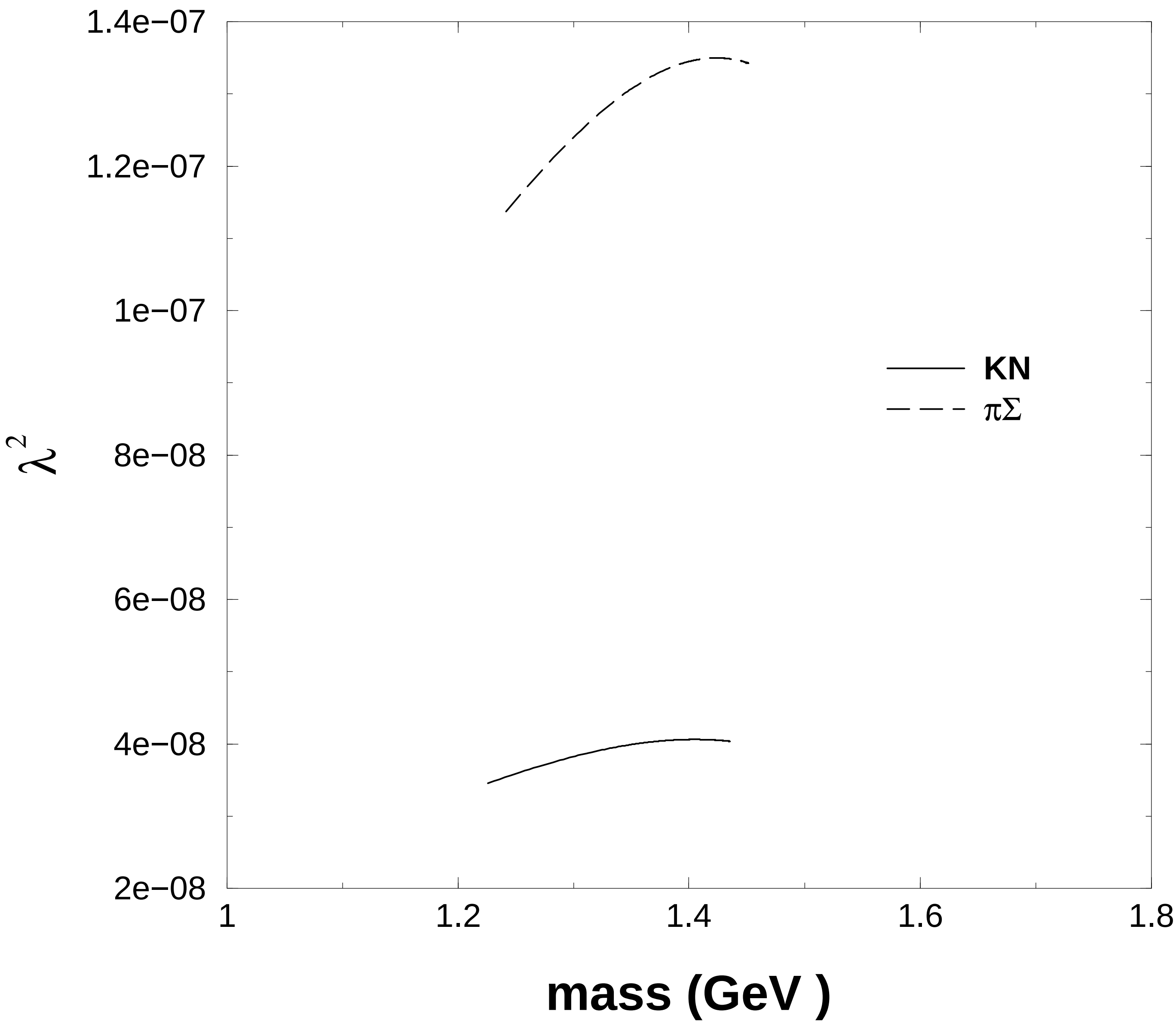
1.7
1.6
1.5
1.4
1.3
1.2
0
2
4
6
8
10
mass (GeV)
M^2 (GeV^2)
With SU(3) sym. breaking effects
W/o SU(3) sym. breaking effects

Fig. 8



1.7
1.6
1.5
1.4
1.3
1.2
mass (GeV)
0
2
4
6
8
10
M^2 (GeV^2)
"unbound" + "bound" diagrams
"unbound" diagrams only

Fig. 9



1.4e−07
1.2e−07
1e−07
8e−08
6e−08
4e−08
2e−08
1
1.2
1.4
1.6
1.8
KN
πΣ
λ2
mass (GeV)